"Anything worth measuring is worth measuring well."

Source unknown

Mathematics & Measurements

To determine if a runner broke a world's record in a sprint or marathon, the time that passed between the start and finish must be carefully measured and compared to the world records. Since time can be measured and expressed as an amount, it is called a **quantity**. Ten seconds, two minutes, and five hours are examples of quantities of time. Other familiar quantities that are important in chemistry include mass (similar to the more familiar *weight*), length, volume, temperature, and density.

The International System of Units

In 1960, a group of scientists from many fields and many countries agreed upon a set of metric units that would serve as a standard for scientific communication. This standard set of units is known as the **International System of Units** and is abbreviated **SI** (the abbreviation is derived from the French spelling *le Systeme International d' Unites*). Seven quantities are the foundation for SI, and each has a **base unit** in which that quantity is expressed. Table 1 lists the base units for length, mass, volume, temperature, time and chemical amount, along with their abbreviations and their relationships to common United States units.

Table 1.

Quantity	***U.S.***	***SI Base Unit***	***Chemistry***
Mass (weight)	Pound (lb)	Kilogram (kg)	"Gram" (**g**, mg)
Volume	Gallon (gal)	Liter (L)	"Liter" (**mL**, L)
Temperature	Fahrenheit (^{o}F)	Kelvin (K)	**K** & Celsius (**^{o}C**)
Length	Mile (mi), Feet(ft), Inches (in)	Meter (m)	"Meter" (**cm**, mm, nm)
Time	Second (s)	Second (s)	Second (s)
			Mole (mol)

SI Base Units Equivalents

Quantity	***Base Unit***	***Abbreviation***	***U.S. Equivalent***
Mass	kilogram	kg	2.205 pounds
Volume	liter	L	0.946 quarts
Length	meter	m	39.37 inches

The three SI base units for mass, volume and length in Table 1 were chosen because they correspond to magnitudes that are convenient for everyday measurement. They are well suited for measurements on a scale that we can directly relate to. However, chemists often work with very small quantities such as those used to express the diameter of a hydrogen atom or huge quantities such as the number of particles in a kilogram of carbon. These numbers are beyond the range of our senses and cannot be conveniently expressed in standard notation in SI units. Thus, the system of scientific notation is used to express very small and very large quantities.

Scientific notation is a method of expressing numbers as a product of two factors. The first factor is a number that is greater than or equal to 1 but less than 10. The second factor is 10 raised to a power. The power of 10, or exponent, is positive for numbers greater than 10 and negative for numbers less than 1. Table 2 gives examples showing both the ordinary decimal form and the exponential form for some quantities. Notice how scientific notation eliminates the need to write a long list of zeros in very small and very large numbers.

Table 2. Examples of Quantities Expressed in Scientific Notation

Quantity	Ordinary Decimal Form	Scientific Notation
Diameter of a hydrogen atom	0.000 000 000 074 m	7.4×10^{-11} m
Mass of a hydrogen atom	0.000 000 000 000 000 000 000 000 11 kg	1.1×10^{-25} kg
Number of molecules in 2.0 g of hydrogen	600 000 000 000 000 000 000 000	6.0×10^{23} molecules

Metric Prefixes To further simplify the expression of measured quantities, scientists use prefixes with metric units to represent powers of ten. The following Tables list metric prefixes with a range of 40 orders of magnitude including those frequently used in chemistry. Notice that each prefix has an abbreviation and an equivalent power of ten. You will use those prefixes in bold face italics and their associated powers of ten listed in Table 3. If you can't remember or access them, you will need to memorize them.

Table 3. Commonly used prefixes and Equivalent Powers of Ten. Know the bold italics.

Prefix	Abbreviation	Power of Ten	Example
pico-	p	10^{-12}	picogram, pg
nano-	n	10^{-9}	***nanometer, nm***
micro-	μ	10^{-6}	microsecond, μs
milli-	m	10^{-3}	***milliliter, mL***
centi-	c	10^{-2}	***centimeter, cm***
kilo-	k	10^{3}	***kilometer, km***
mega-	M	10^{6}	***megahertz, MHz***
giga-	G	10^{9}	***gigabyte, GB***
hella-	H	10^{27}	*The universe*

Measured Quantities and Significant Figures

There is a degree of uncertainty in every measurement, and this uncertainty, by convention, is reflected in the last recorded digit of any measured quantity. If several people measure the same distance with a ruler, their measurements will probably differ in the last digit. However, these measurements will cluster around the true value. Some will be equal to the true value, some will be higher, and some will be lower. The average of a series of measurements is generally considered to represent the most accurate value for the entire, complete series of data measured.

The number of **significant figures** in a single measurement or average of the measurements is the total of the number of digits known with certainty plus one uncertain digit. The digits known with certainty are those that can be read very precisely from the measuring instrument. The last digit recorded in any measured quantity or average—the uncertain digit—is estimated.

When a measurement is written in scientific notation, the first factor represents the significant digits in the measured quantity. The second factor, the power of ten, represents the magnitude of the measurement, or the number of decimal places, and therefore has nothing to do with the number of significant figures in the quantity. If the three-significant-figure measured quantity 134 pounds is written in scientific notation as 1.34×10^2 pounds, the number of significant figures cannot change, so this must also have three significant figures.

The following rules summarize the conventions used by chemists when working with measured quantities:

1. The last digit expressed in a measured quantity is the uncertain, or estimated, digit.

2. Zeros that serve as place holders in ordinary decimal notation are not significant. For example, the measured quantity 0.001 m has **one** significant figure (s.f.). This becomes more apparent in the quantity when written in scientific notation: **one** s.f.: 1×10^{-3} m; **two** s.f.: 0.0010 (1.0×10^{-3} m); **three** s.f.: 0.00100 (1.00×10^{-3} m);

3. When adding and subtracting numbers representing measured quantities, the number of decimal places in the sum or difference are limited by the number of decimal places in the measured quantity that has the least number of decimal places. For example, 0.152 g + 0.26 g = 0.41 g.

4. When multiplying and/or dividing numbers that represent measured quantities, add the powers of ten when multiplying, and subtract when dividing. The number of significant figures in the result (first factor) is the smallest number of significant figures in the measured number (first factor) with the least number of significant figures in all of the numbers multiplied or divided. For example, $(1.5 \times 10^2\ \text{m}) \times (1.350 \times 10^3\ \text{m})$ x $\times (1.000000000 \times 10^0\ \text{m}) = 2.0 \times 10^5\ \text{m}^2$.

Mathematical Operations in Scientific Notation (*Calculator Key Strokes*)

Addition and Subtraction To add or subtract numbers that are expressed in scientific notation, use the "enter exponent" key on your calculator to express the power of ten. It is usually labeled "EE" or "EXP." *Do not use the 10 ^{x} or y ^{x} key for scientific notation exponents.* The calculator can express the result in scientific notation automatically. For example:

3.25×10^3 mL + 4.66×10^4 mL =

Calculator key sequence: $\boxed{3}$ $\boxed{.}$ $\boxed{2}$ $\boxed{5}$ $\boxed{\text{EE}}$ $\boxed{3}$ $\boxed{+}$ $\boxed{4}$ $\boxed{.}$ $\boxed{6}$ $\boxed{6}$ $\boxed{\text{EE}}$ $\boxed{4}$ $\boxed{=}$

Result: 4.985×10^4 mL

Multiplication To multiply numbers that are expressed in scientific notation, use your calculator. For example:

(3.1×10^2) m $\times$ (5.2×10^4) m =

Calculator key sequence: $\boxed{3}$ $\boxed{.}$ $\boxed{1}$ $\boxed{\text{EE}}$ $\boxed{2}$ $\boxed{\times}$ $\boxed{5}$ $\boxed{.}$ $\boxed{2}$ $\boxed{\text{EE}}$ $\boxed{4}$ $\boxed{=}$

Result: $1.6\underline{12\ldots} \times 10^7$ m^2

Division To divide numbers that are expressed in scientific notation, again, let your calculator do the work. For example:

$$\frac{7.5 \text{ x } 10^2 \text{ g}}{5.9 \text{ x } 10^{-4} \text{ cm}^3} =$$

Calculator key sequence: $\boxed{7}$ $\boxed{.}$ $\boxed{5}$ $\boxed{\text{EE}}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{.}$ $\boxed{9}$ $\boxed{\text{EE}}$ $\boxed{4}$ $\boxed{^+/_-}$ $\boxed{=}$

Result: $1.2\underline{71}\ldots \times 10^6$ g/cm^3

Powers To raise a number expressed in scientific notation to a power, use the y^x key on your calculator. For example:

$(2.5 \times 10^2 \text{ cm})^3$ =

Calculator key sequence: $\boxed{2}$ $\boxed{.}$ $\boxed{5}$ $\boxed{\text{EE}}$ $\boxed{2}$ $\boxed{y^x}$ $\boxed{3}$ $\boxed{=}$

Result: $1.5\underline{6}\ldots \times 10^7$ cm^3